

UNIT 10

Algebra

Telecommunication companies offer telephone services.

These tables show the plans for cell phones for two companies. Each plan includes 200 free minutes.

What patterns do you see in the tables?

Write a pattern rule for each pattern. Describe each plan.

Assume the patterns continue. How could you find the total cost for 60 additional minutes for each plan?

Company A

Number of Additional Minutes	Total Cost (\$)
0	35
4	36
8	37
12	38
16	39
20	40

Company B

Number of Additional Minutes	Total Cost (\$)
0	40
5	41
10	42
15	43
20	44
25	45

What You'll Learn

- Investigate number properties.
- Write an expression for the n th term of a pattern.
- Evaluate algebraic expressions by substituting fractions and integers.
- Read, write, and solve equations.
- Represent algebraic relationships using tables, graphs, and equations.

Why It's Important

- Algebra is used to communicate with symbols. It can be used to describe patterns.
- Patterns and equations are used to investigate changes in our world. For example, urban planners use equations to investigate population growth.



Key Words

- distributive property
- expand

Herro Sam!

Herro Sam!

Report of the

Writing Expressions and Equations

We use a letter, such as x or n , to represent a number.

We can write an algebraic expression to represent a word statement.

For example, “a number plus five,” or “five more than a number” can be written as $n + 5$.

When we write an algebraic expression as equal to a number or another expression, we have an equation.

For example, $n + 5 = 8$ is an equation.

Example 1

- a) Write an algebraic expression for this statement:
Three more than four times a number
- b) Write an equation for this sentence:
A number divided by four is 5.

Solution

- a) Three more than four times a number
Let x represent the number.
Then, four times a number is $4x$.
Three more than $4x$ is:
 $4x + 3$ or $3 + 4x$

- b) A number divided by four is 5.
Let z represent the number.
 z divided by four is: $\frac{z}{4}$
The equation is: $\frac{z}{4} = 5$

✓ Check

- Write an algebraic expression for each statement.
 - a number multiplied by seven
 - six less than a number
 - five more than three times a number
 - three less than five times a number
- Write an equation for each sentence.
 - A number divided by seven is 6.
 - The sum of eight and a number is 17.
 - Five more than two times a number is 11.



Evaluating Expressions

To evaluate an algebraic expression for a particular value of the variable, replace the variable with a number. Then, find the value of the expression. The number we substitute can be a fraction or an integer.

Example 2

Evaluate the expression $2x + 3y + 4z$ for $x = -1$, $y = \frac{1}{3}$, and $z = \frac{1}{2}$.

Solution

$$2x + 3y + 4z$$

Substitute: $x = -1$, $y = \frac{1}{3}$, and $z = \frac{1}{2}$

$$\begin{aligned} 2x + 3y + 4z &= 2(-1) + 3\left(\frac{1}{3}\right) + 4\left(\frac{1}{2}\right) \\ &= 2 \times (-1) + 3 \times \frac{1}{3} + 4 \times \frac{1}{2} && \text{Multiply first.} \\ &= -2 + 1 + 2 && \text{Then add.} \\ &= -2 + 3 \\ &= 1 \end{aligned}$$

✓ Check

3. Evaluate each expression.

a) $3 + x$ for $x = \frac{1}{2}$ b) $3 - x$ for $x = -2$ c) $3x$ for $x = \frac{1}{4}$

4. Evaluate each expression for $p = \frac{2}{3}$ and $q = \frac{1}{4}$.

a) $p + q$ b) $p - q$ c) pq

5. Evaluate each expression for $m = \frac{2}{5}$ and $n = \frac{1}{2}$.

a) $2m + n$ b) $2n + m$ c) $2m + 2n$
d) $2m - n$ e) $2n - m$ f) $2n - 2m$
g) mn h) $2mn$ i) $\frac{1}{2}mn$

6. Evaluate each expression in question 5 for $m = -3$ and $n = -6$.

7. Evaluate each expression.

a) $3x - 2y + 4z$, when $x = \frac{3}{4}$, $y = \frac{1}{5}$, $z = \frac{5}{4}$
b) $3x + 5y - 3z$, when $x = \frac{5}{4}$, $y = \frac{1}{6}$, $z = \frac{2}{3}$
c) $3x + 3y - 2z$, when $x = \frac{1}{5}$, $y = \frac{4}{3}$, $z = \frac{1}{15}$

8. Evaluate each expression in question 7 for $x = 2$, $y = -4$, and $z = -1$.

Focus Relate the distributive property and other properties to algebra.

Recall how you used a diagram to multiply: 4×37

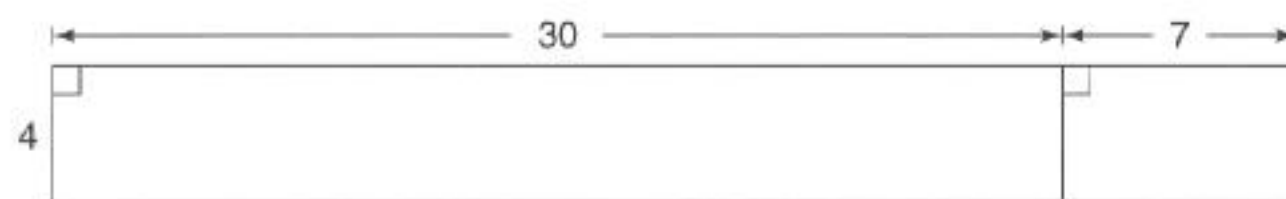
This diagram shows:

$$4 \times 37 = 4 \times (30 + 7)$$

$$= 4 \times 30 + 4 \times 7$$

$$= 120 + 28$$

$$= 148$$



Explore

$5(n + 8)$ means
 $5 \times (n + 8)$.

Work with a partner. Use 0.5-cm grid paper if it helps.

- Draw a diagram to illustrate 5×28 .
What is the product?
- Draw a diagram to illustrate $5(n + 8)$.
What is the product?
- Draw a diagram to illustrate $5(n + m)$.
What is the product?
- Draw a diagram to illustrate $d(n + m)$.
What is the product?

Reflect & Share

Compare your diagrams and products with another pair of classmates.

What patterns do you see in the products?

How can you use the patterns to write $d(n + m)$ without brackets?

Connect

When we use symbols to represent numbers, the following properties are still true.

Adding 0

Adding 0 does not change the number.

$$4 + 0 = 4 \text{ and } n + 0 = n$$

$$0 + 135 = 135 \text{ and } 0 + n = n$$

Multiplying by 1

When 1 is a factor, the product is always the other factor.

$$1 \times 11 = 11 \text{ and } 1 \times n = n$$

$$256 \times 1 = 256 \text{ and } n \times 1 = n$$

Multiplying by 0

When 0 is a factor, the product is always 0.

$$15 \times 0 = 0 \text{ and } n \times 0 = 0$$

$$0 \times 137 = 0 \text{ and } 0 \times n = 0$$

Order of addition and multiplication

When you add, the order does not matter.

$$9 + 4 = 13 \text{ and } 4 + 9 = 13 \quad a + b = b + a$$

When you multiply, the order does not matter.

$$6 \times 8 = 48 \text{ and } 8 \times 6 = 48 \quad ab = ba$$

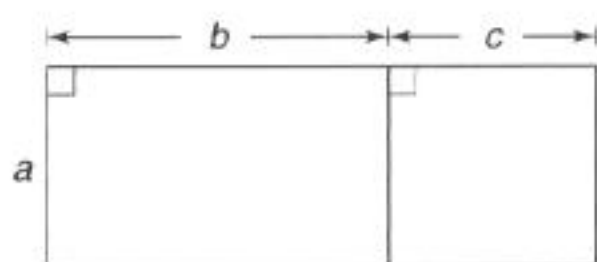
Distributive Property

We will investigate $a(b + c)$ and $ab + ac$ for different values of a , b , and c .

Recall that $a(b + c)$ means $a \times (b + c)$, ab means $a \times b$, and ac means $a \times c$.

a	b	c	$(b + c)$	$a(b + c)$	ab	ac	$ab + ac$
2	4	7	11	22	8	14	22
3	6	2	8	24	18	6	24
7	1	1	2	14	7	7	14
12	8	3	11	132	96	36	132
0	7	5	12	0	0	0	0

We can illustrate this property with a diagram.



The numbers in these columns are the same.

This table illustrates the **distributive property** of multiplication:

$$a(b + c) = ab + ac$$

That is, the product of $a(b + c)$ is the same as the sum $ab + ac$.

Example

Use the distributive property to write each expression as a sum of terms.

a) $7(c + 2)$

b) $2(2a + 3b + 4)$

Solution

$$\begin{aligned} \text{a) } 7(c + 2) &= 7(c) + 7(2) \\ &= 7c + 14 \end{aligned}$$

$$\begin{aligned} \text{b) } 2(2a + 3b + 4) &= 2(2a) + 2(3b) + 2(4) \\ &= 4a + 6b + 8 \end{aligned}$$

In the *Example*, when we use the distributive property, we **expand**.

Practice

1. Draw a rectangle to show that $5(x + 2)$ and $5x + 10$ are equivalent.

2. Expand.

a) $2(x + 10)$

b) $5(x + 1)$

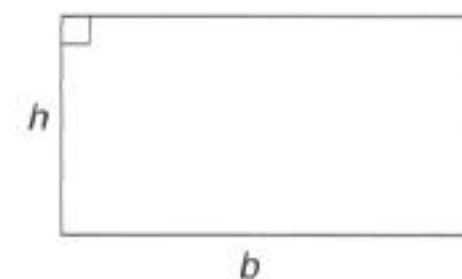
c) $10(x + 2)$

d) $6(12 + 6y)$

e) $8(8 + 9y)$

f) $5(7y + 6)$

3. Write two formulas for the perimeter, P , of a rectangle. Explain how the formulas illustrate the distributive property.



4. Explain how you know $hb = bh$. Use an example to justify your answer.

5. Expand.

a) $5(2x + 2y + 2)$

b) $4(3x + 5y + 1)$

c) $8(7x + 3y + 2)$

6. **Assessment Focus** Which expressions in each pair are equivalent? Explain your reasoning.

a) $2x + 20$ and $2(x + 20)$

b) $3x + 7$ and $10x$

c) $6 + 2t$ and $2(t + 3)$

d) $9 + x$ and $x + 9$

Calculator Skills

Instead of multiplying a number by 43.2, a student divided it by 43.2.

The student got the answer 35. What should the answer have been?

Reflect

What is the distributive property?
Include a diagram with your explanation.

Focus Write an expression for the n th term of a number pattern.

Explore



Work with a partner.

You will need grid paper.

Charla has juvenile diabetes.

She needs five injections of insulin per day.

Each needle can be used only once.

Charla wants to go to camp.

She must take all her needles with her.

She must always have at least 6 extra needles available.

- Copy and complete this table.
Find the number of needles Charla needs to take with her for up to 6 days.
- Graph the data.
- Write an algebraic expression for the number of needles required for any number of days.
Use the expression to find the number of needles required for 7 days, 14 days, and 30 days.

Number of Days	Number of Needles
1	
2	
3	
4	
5	
6	

Reflect & Share

Compare your results with those of another pair of classmates. Work together to explain how the table, the graph, and the expression are related.

Connect

We can use a table, a graph, and algebra to describe and extend a number pattern.

Look at the pattern: 1, 3, 5, 7, ...

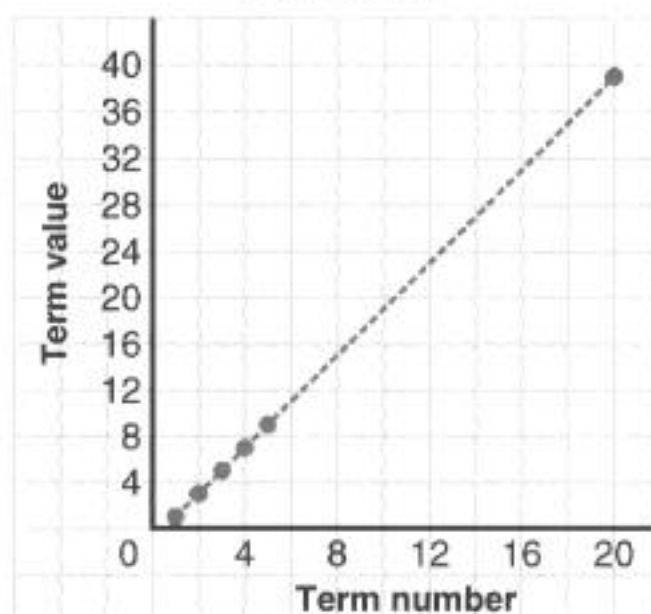
To find the 20th term, use one of these three methods.

- Make a table, then extend the table to find the 20th term.
The term value increases by 2 each time.
The pattern rule is: Start at 1. Add 2 each time.
From the table on the next page, the 20th term is 39.

Term Number	Term Value
1	1
2	3
3	5
4	7
5	9
20	39

The 20th term is 39.

Graph of Number Pattern
1, 3, 5, 7, ...



➤ Graph the pattern, then extend the graph to find the 20th term.

The points lie on a straight line.

To get from one point to another, move 1 unit right and 2 units up.

Use a ruler to draw a broken line through the points to show the trend.

Extend the broken line to the right to find that the 20th term is 39.

➤ The term values are consecutive odd numbers: 1, 3, 5, 7, 9, ...
The algebraic expression $2n$ produces even numbers, when we substitute $n = 1, 2, 3, 4, \dots$

That is, $2(1) = 2$

$$2(2) = 4$$

$$2(3) = 6$$

$$2(4) = 8, \text{ and so on}$$

Each odd number is 1 less than the following even number.

So, the expression $2n - 1$ produces odd numbers,

when we substitute $n = 1, 2, 3, 4, \dots$

That is, $2(1) - 1 = 2 - 1 = 1$

$$2(2) - 1 = 4 - 1 = 3$$

$$2(3) - 1 = 6 - 1 = 5$$

$$2(4) - 1 = 8 - 1 = 7$$

This table shows how the term value relates to the term number.

Term Number	Term Value	Pattern Rule for Term Value
1	1	$1 = 2(1) - 1$
2	3	$3 = 2(2) - 1$
3	5	$5 = 2(3) - 1$
4	7	$7 = 2(4) - 1$
5	9	$9 = 2(5) - 1$

In each case, the term value is equal to:
The term number multiplied by 2, then subtract 1

The natural numbers are the counting numbers: 1, 2, 3, ...

Let t represent the term number.

Then an expression for the term value is $2t - 1$, where t is any natural number.

To check that the expression for the term value is correct, substitute a number for t .

Substitute $t = 2$.

$$\begin{aligned}2t - 1 &= 2 \times 2 - 1 \\ &= 4 - 1 \\ &= 3\end{aligned}$$

So, the 2nd term is 3, which matches the 2nd term in the pattern given.

This method allows us to find the value of any term in the pattern.

For example, the 20th term has value: $2(20) - 1 = 39$

Example

Here is a number pattern.

8, 12, 16, 20, ...

- Complete a table for the first 5 terms of this pattern. Extend the table to find the 10th term. Describe the pattern. Write a pattern rule.
- Graph the pattern.
- Write an expression for the n th term.
- Use the expression in part c to verify the 10th term.

Solution

- 8, 12, 16, 20, ...

The pattern begins with 8.

To get the next term, add 4 each time.

The pattern rule is: Start at 8. Add 4 each time.

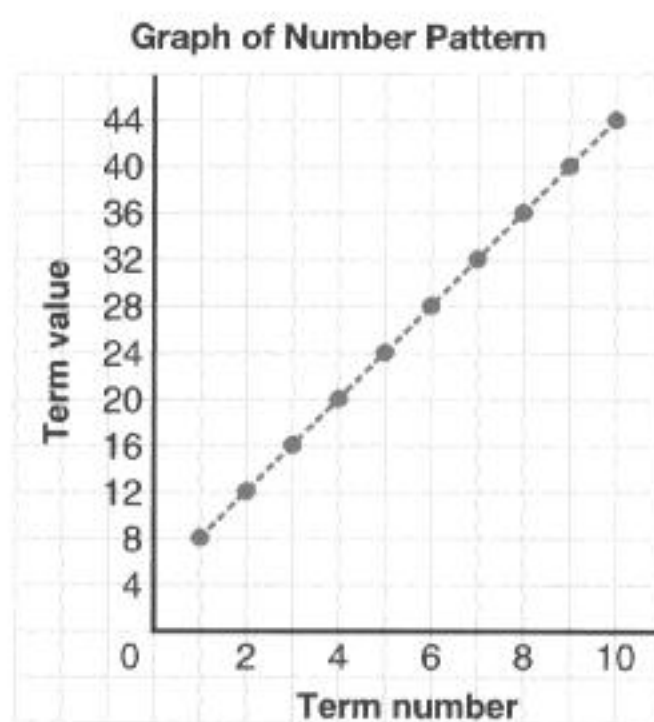
Extend the table to find the 10th term is 44.

Term Number	1	2	3	4	5	6	7	8	9	10
Term Value	8	12	16	20	24	28	32	36	40	44

- Graph the pattern.

The points lie on a straight line.

Use a ruler to draw a broken line through the points to show the trend.



- c) Find a pattern rule that relates the term value to the term number.

Each term is 4 more than the previous term.

Look for patterns that involve multiples of 4.

In each case, the term value is equal to:
Four more than
four times the
term number

Term Number	Term Value	Pattern Rule for Term Value
1	$8 = 4 + 4$	$8 = 4(1) + 4$
2	$12 = (4 + 4) + 4$	$12 = 4(2) + 4$
3	$16 = (4 + 4 + 4) + 4$	$16 = 4(3) + 4$
4	$20 = (4 + 4 + 4 + 4) + 4$	$20 = 4(4) + 4$

To write an expression for the n th term, let n represent any term number. Then, the n th term is: $4n + 4$

- d) To find the 10th term, substitute $n = 10$ into $4n + 4$.

$$\begin{aligned} 4n + 4 &= 4(10) + 4 \\ &= 44 \end{aligned}$$

The 10th term is 44. This verifies the value in the table in part a.

Practice

- Substitute $n = 1, 2, 3, 4, 5,$ and 6 to generate a number pattern. Describe each pattern, then write a pattern rule.
 - $2n + 1$
 - $3n - 1$
 - $2n + 2$
 - $4n - 2$
- For each number pattern, write an expression for the n th term.
 - $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$
 - $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$

Number Strategies

The product of two fractions is $\frac{1}{2}$.

Find four different pairs of fractions that have a product of $\frac{1}{2}$.

3. For each number pattern below:
- Describe the pattern. Write the pattern rule.
 - Use a table to find the 12th term.
 - Write an expression for the n th term.
 - Use the expression to find the 100th term.
- a) 1, 2, 3, 4, 5, ... b) 2, 3, 4, 5, 6, ...
c) 3, 4, 5, 6, 7, ... d) 4, 5, 6, 7, 8, ...
4. For each number pattern below:
- Write a pattern rule. Justify your rule.
 - Graph the pattern. Use the graph to find the 9th term.
 - Write an expression for the n th term.
 - Use the expression to find the 60th term.
- a) 2, 4, 6, 8, 10, ... b) 6, 9, 12, 15, 18, ...
c) 3, 7, 11, 15, 19, ... d) 10, 15, 20, 25, 30, ...
5. Here are two number patterns.
- 1, 4, 9, 16, 25, ...
 - 4, 8, 16, 32, 64, ...
- Does the number 512 appear in either pattern? Both patterns? Justify your answer.
6. **Assessment Focus** Here is the beginning of a number pattern.
10, 20, ...
- Extend the pattern in two different ways.
 - Describe each pattern. Write a pattern rule for each.
 - Write an expression for the n th term for one pattern.
 - Can you write an expression for the n th term of the other pattern? Explain.
7. For each number pattern below:
- Write a pattern rule. Justify your rule.
 - Find the 15th term.
 - Write an expression for the n th term.
 - Use the expression to find the 30th term.
- a) $\frac{2}{2}, \frac{3}{5}, \frac{4}{8}, \frac{5}{11}, \dots$ b) 1, 3, 6, 10, 15, ...

Take It Further

Reflect

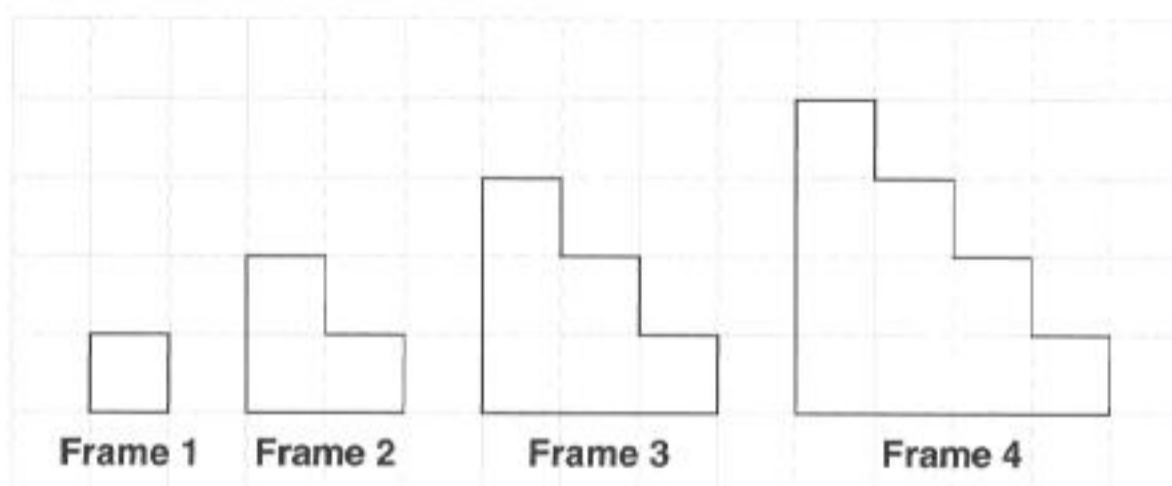
Name three ways to describe and extend a number pattern. Which way is the most efficient? Explain.

Focus Write an expression for the n th term of a geometric pattern.

Explore



Work in a group.
Here is a pattern with squares.



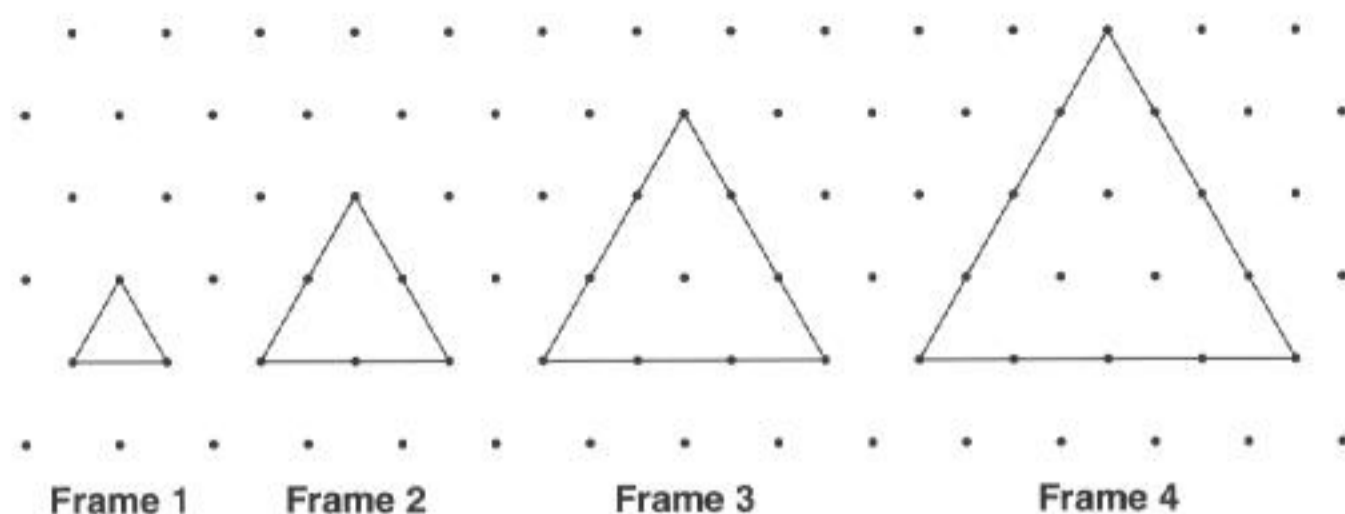
The pattern continues.
Find the perimeter of each frame.
What pattern do you see in the perimeters?
Use a table to show the pattern.
Graph the pattern.
Write a rule for the pattern.
Use a variable.
Write an algebraic expression you could use to find the perimeter of any frame.
Use the expression to find the perimeters of Frame 5, Frame 10, and Frame 100.

Reflect & Share

Share your algebraic expression with that of another group.
Are the expressions the same?
If not, how can you check if either expression is correct?
Could both expressions be correct? Explain.

Connect

We can use algebra to describe and extend a geometric pattern.
Here is a pattern of equilateral triangles drawn on isometric paper.



Frame	Perimeter (units)
1	3
2	6
3	9
4	12

This table shows the perimeter of each frame.

The pattern rule for the perimeters is:

Start at 3. Add 3 each time.

If we use this pattern rule to find the perimeter of Frame 40, we would need to know the perimeter of all the frames from Frame 1 to Frame 39.

Instead, we look for a pattern rule for the perimeter in terms of the frame number.

The perimeters are multiples of 3, so write each perimeter as a product, with one factor of 3.

Frame	Perimeter (units)	Perimeter as a Product
1	3	$3 = 3 \times 1$
2	6	$6 = 3 \times 2$
3	9	$9 = 3 \times 3$
4	12	$12 = 3 \times 4$

In each case, the perimeter is equal to 3 times the frame number.

We can use this pattern to find the perimeter of Frame 40:

$$3 \times 40 = 120$$

The perimeter of Frame 40 is 120 units.

We write the pattern using algebra.

Let f represent the frame number.

Then, an algebraic expression for the perimeter of Frame f is: $3f$
 f is any natural number.

To check that the expression is correct, substitute a number for f .

Substitute $f = 4$.

$$\begin{aligned} 3f &= 3(4) \\ &= 12 \end{aligned}$$

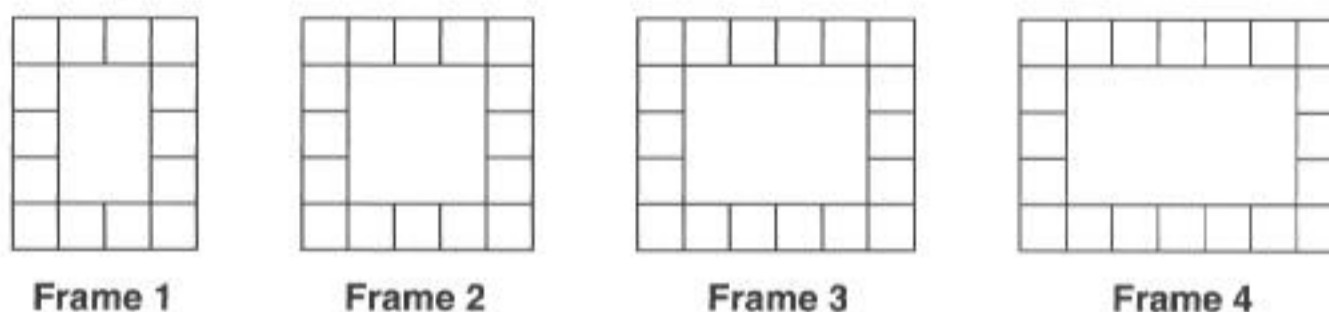
So, Frame 4 has perimeter 12 units.

This is verified by the table on page 429.

Example

Picture frames are decorated with square tiles in the pattern shown. Each tile has side length 1 cm.

The pattern continues.



a) Find the area of the picture in each frame.

What pattern do you see in the areas?

b) Graph the pattern in part a.

How does the graph illustrate the pattern?

c) Use a variable.

Write an algebraic expression for the area of the picture in any frame.

d) Use the expression in part c.

Find the area of the picture in Frame 99.

Solution

a) Each picture is a rectangle.

Its area is length \times width.

Write the areas in a table.

The areas are multiples of 3.

The pattern rule is:

Start at 6. Add 3 each time.

b) The graph starts at (1, 6).

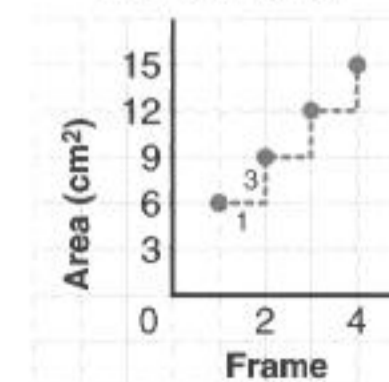
To get the next point each time, move 1 right and 3 up.

Moving 1 right is the increase in the frame number.

Moving 3 up is the increase in the area.

Frame	Area of Picture (cm ²)
1	$3 \times 2 = 6$
2	$3 \times 3 = 9$
3	$3 \times 4 = 12$
4	$3 \times 5 = 15$

Area of Picture against Frame Number



- c) For an algebraic expression, look at each area in terms of the frame number.

Adding 3 each time indicates a pattern where the term number is multiplied by 3.

So, multiply each term number by 3 and find out what needs to be added each time to get the area.

Frame	Area of Picture (cm ²)	Area in Terms of Frame Number
1	6	$3 \times 1 + 3$
2	9	$3 \times 2 + 3$
3	12	$3 \times 3 + 3$
4	15	$3 \times 4 + 3$

Each area is: 3 times the frame number, then add 3

Use the variable n .

An algebraic expression for the area of the picture in Frame n is:
3 times n , then add 3

This is written: $3n + 3$

- d) For the area of the picture in Frame 99,
substitute $n = 99$ in $3n + 3$.

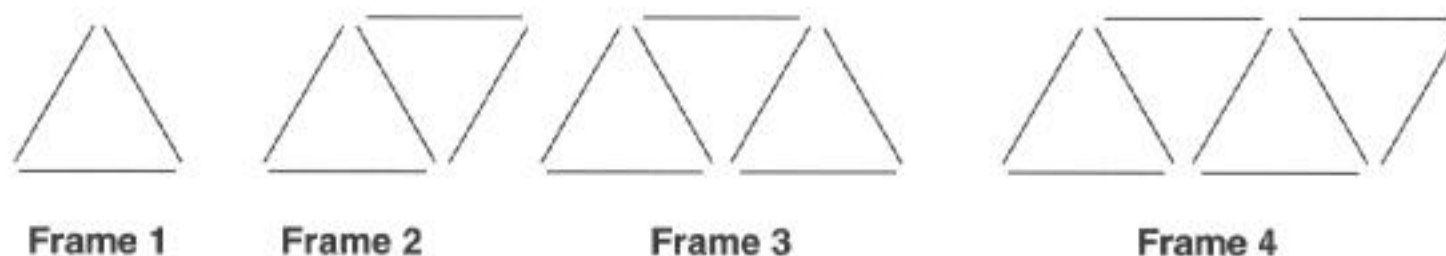
$$\begin{aligned}3n + 3 &= 3(99) + 3 \\ &= 297 + 3 \\ &= 300\end{aligned}$$

The picture in Frame 99 has area 300 cm².

Practice

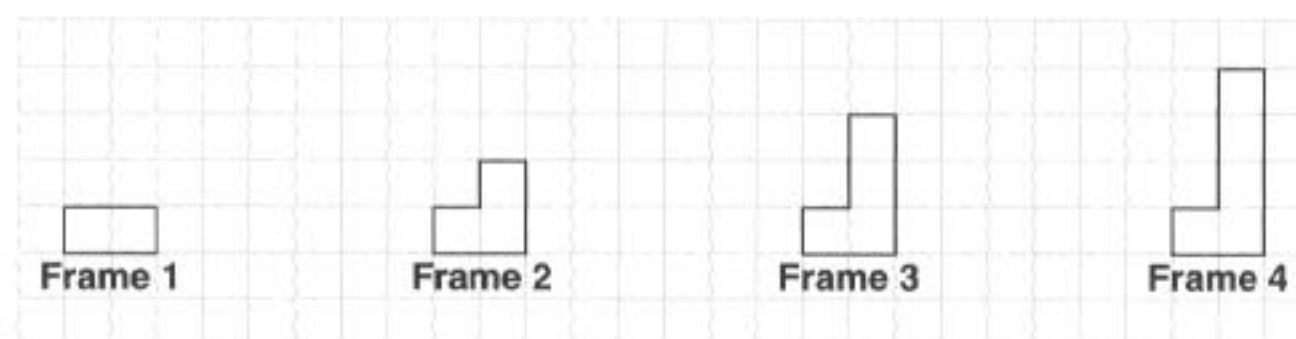
- Use the pattern of frames in the *Example*.
Each frame has the same height of 5 cm.
 - Find the length of each frame.
Make a table.
What patterns do you see in the lengths?
 - Graph the pattern.
How does the graph illustrate the pattern?
 - Write an algebraic expression for the length of the n th frame.
 - Use the expression in part c.
Find the length of Frame 50.

2. Here is a pattern of triangles made with congruent toothpicks.



The pattern continues.

- Find the number of toothpicks in each frame.
What patterns do you see?
 - Graph the data in part a.
 - Write an algebraic expression for the number of toothpicks in the n th frame.
 - Find the number of toothpicks in Frame 45.
3. Here is a pattern of squares.



Each square has side length 1 cm.

The pattern continues.

- Find the perimeter of each frame. Make a table.
What pattern do you see in the perimeters?
 - Graph the pattern. Explain how the graph illustrates the pattern.
 - Write an algebraic expression for the perimeter of the n th frame.
 - Find the perimeter of Frame 75.
4. Here is a pattern made from congruent square tiles.

Each tile has side length 1 cm.

The pattern continues.



- Find the area of each frame.
What patterns do you see in the areas?
- Use a pattern to find the area of Frame 8.
- Write an algebraic expression for the area of the n th frame.
- Which frame has an area of 625 cm^2 ? Justify your answer.

Number Strategies

The six-digit number 63_751 is divisible by 9.

Which digit can go in the thousands place?

How many answers can you find?

Recall that a growing pattern is a pattern that grows in a predictable way.

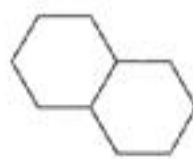
Take It Further



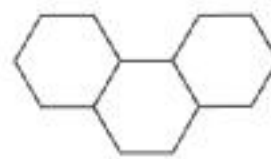
5. Hexagonal tables are arranged as shown below.
One person sits at each side of a table.
The pattern continues.



Frame 1



Frame 2



Frame 3



Frame 4

- How many people can sit at the tables in each frame?
What pattern do you see in the number of people?
- How many people can sit at the tables in Frame 9?
- Explain how you could find the number of people who could be seated at any table arrangement in this pattern.

6. **Assessment Focus** Use grid paper.

- Draw the first four frames of a growing pattern.
- Describe the patterns in the frames.
- Describe or draw Frame 5, Frame 10, and Frame 100.
- Choose one aspect of your pattern;
for example, area, perimeter, and so on.
Write an algebraic expression for the n th frame
of your pattern.

7. Bryn has a sheet of paper. He cuts the paper in half to produce two pieces. Bryn places one piece on top of the other. He then cuts these pieces in half. The pattern continues. The table shows some of the results.

Number of Cuts	1	2	3	4	5	6	7	8	9	10
Number of Pieces	2	4	8							

- Copy and complete this table.
- What patterns do you see in the number of pieces?
- Use a pattern to find the number of pieces after 15 cuts.
- Write an algebraic expression for the number of pieces after n cuts.

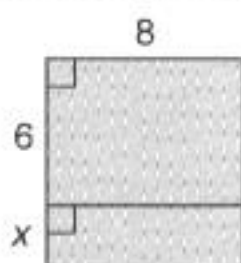
Reflect

Explain the meaning of the term “ n th frame.”

Mid-Unit Review

LESSON

- 10.1 1.** Write two expressions for the area of the shaded rectangle.



- 2.** Draw a rectangle to show that:
 $6(3 + a) = 18 + 6a$

- 3.** Expand.
 a) $3(x + 11)$
 b) $5(12 + y)$
 c) $4(x + 5y + 9)$
 d) $8(5x + 2y + 3)$

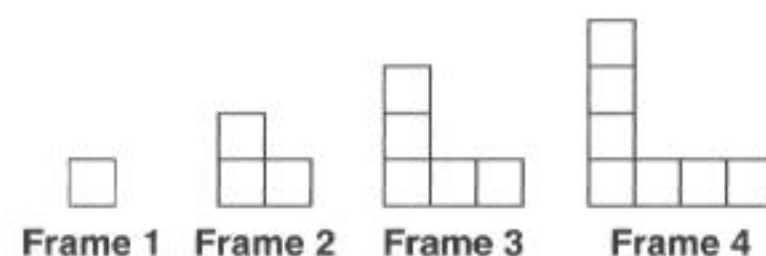
- 10.2 4.** For each number pattern below:

- a) Use a table to find the 8th term.
Describe the pattern.
Write a pattern rule.
- b) Graph the pattern. Use the graph to find the 12th term.
- c) Write an expression for the n th term.
- d) Use the expression to find the 40th term.
 i) 1, 7, 13, 19, 25, ...
 ii) 2, 7, 12, 17, 22, ...
 iii) 4, 7, 10, 13, 16, ...

- 5.** Laurel buys a box of mechanical pencils, and a tube of 8 refill leads. Each pencil contains 3 leads. Laurel puts the tube of refill leads into her pencil case, then adds one pencil at a time.

- a) Make a table to show the number of leads in the pencil case for up to 7 pencils.
Describe the pattern.
Write a pattern rule.
- b) Graph the data in the table.
- c) Write an algebraic expression for the number of leads in the pencil case for any number of pencils.
- d) Use the expression in part c to find the number of leads in the pencil case for 21 pencils.

- 10.3 6.** Here is a pattern made with congruent square tiles.



- a) Count the number of tiles in each frame.
What pattern do you see?
- b) Make a table to show the pattern.
- c) Graph the pattern.
- d) Write an algebraic expression for the number of tiles in the n th frame.
- e) Use the expression in part d to find the number of tiles in Frame 30.
- f) Will any frame have each number of tiles?
 i) 31 ii) 32 iii) 33
How do you know?

Focus Use algebra tiles to solve equations involving whole numbers.

Recall that one red unit tile and one yellow unit tile combine to model 0.

These two unit tiles form a zero pair.



The yellow variable tile represents x .

The opposite of x is $-x$.

So, the red variable tile represents $-x$.



One red variable tile and one yellow variable tile also combine to model 0.

These two variable tiles form a zero pair.

Flip the yellow tile to get a red tile.

Explore

Work with a partner.

You will need algebra tiles.

- For the equation: $2x = 9 - x$
 - Interpret the equation in words.
 - Use algebra tiles to solve the equation.
 - Sketch the tiles you used.

- Repeat the activity for this equation: $2 - 3x = 2x - 8$

Reflect & Share

Compare the solutions for the equations with those of another pair of classmates.

What strategies did you use to solve the equations?

How did you use zero pairs?

Connect

Recall how we used algebra tiles to solve equations in *Unit 1*.

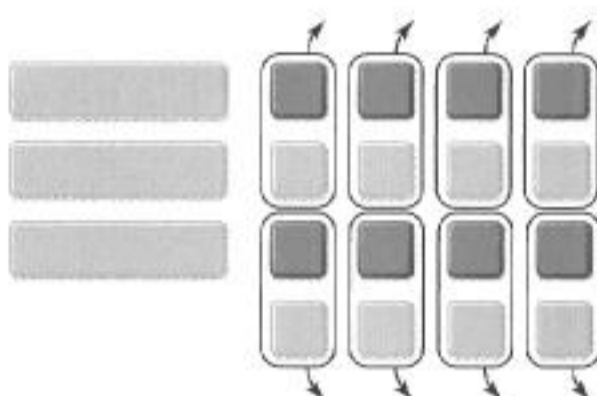
Remember that to keep the balance of an equation, what you do to one side you must also do to the other side.

To solve the equation $3x - 8 = -x$, isolate the variable tiles on one side of the equation.

On the left side, put algebra tiles to represent $3x - 8$.



To isolate the x -tiles on the left side, add 8 yellow unit tiles to make zero pairs.



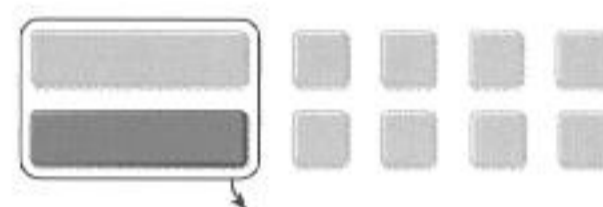
On the right side, put algebra tiles to represent $-x$.



To keep the balance, add 8 yellow unit tiles to this side, too.



To isolate the unit tiles on the right side, add 1 yellow x -tile to each side.



There are 4 x -tiles. So, arrange the unit tiles into 4 equal groups.



The tiles above show the solution $x = 2$.

When you solve an equation, you should always verify the solution. To do this, substitute the solution into the equation to check that it satisfies the equation. Substitute $x = 2$ into $3x - 8 = -x$.

$$\begin{aligned} \text{Left side} &= 3x - 8 \\ &= 3(2) - 8 \\ &= 6 - 8 \\ &= -2 \end{aligned}$$

$$\begin{aligned} \text{Right side} &= -x \\ &= -2 \end{aligned}$$

Since the left side equals the right side, $x = 2$ is correct.

Example

- Use algebra tiles to solve the equation $2x + 3 = 4x - 3$.
- Verify the solution.
- Interpret the equation in words.

Solution

a) $2x + 3 = 4x - 3$



Isolate the x -tiles on the left side.

Add 3 red unit tiles to each side.



Isolate the unit tiles on the right side.

Add 4 red x -tiles to each side.



There are 2 x -tiles. So, arrange the unit tiles into 2 equal groups.



The tiles show that one red x -tile equals 3 red unit tiles.

Flip the tiles on each side.

One yellow x -tile equals 3 yellow unit tiles.

So, $x = 3$

- b) To verify the solution, substitute $x = 3$ into $2x + 3 = 4x - 3$.

$$\begin{aligned}\text{Left side} &= 2x + 3 \\ &= 2(3) + 3 \\ &= 6 + 3 \\ &= 9\end{aligned}$$

$$\begin{aligned}\text{Right side} &= 4x - 3 \\ &= 4(3) - 3 \\ &= 12 - 3 \\ &= 9\end{aligned}$$

Since the left side equals the right side, $x = 3$ is correct.

c) $2x + 3 = 4x - 3$

This means: two times a number plus three is equal to four times the number minus three.

The *Example* shows what you do if you end up with red variable tiles.

Flip the tiles on both sides of the equation.

Practice

1. Interpret each equation in words.

Then use algebra tiles to solve the equation.

a) $2x = x + 5$

b) $3x - 2 = x$

c) $7x - 9 = 4x$

d) $6 - x = 2x$

2. Use algebra tiles to solve each equation.

a) $7 - 3x = -4x + 13$

b) $4x + 3 = 2x + 7$

c) $3x - 4 = x + 2$

d) $5 - x = 7 - 2x$

3. a) Interpret each equation in words.

b) Use algebra tiles to solve each equation.

c) Verify each solution.

i) $2x + 2 = 3x - 5$

ii) $5x - 6 = 8 - 2x$

iii) $3x - 13 = x - 7$

4. One less than two times a number is equal to three more than the number.

Let x represent the number.

Then, an equation is: $2x - 1 = x + 3$

Use algebra tiles to solve the equation. What is the number?

5. Five times a number is equal to two more than three times the number.

Let n represent the number.

Then, an equation is: $5n = 2 + 3n$

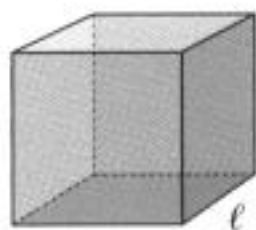
a) Use algebra tiles to solve the equation. What is the number?

b) Verify your solution.

Calculator Skills

The Rowan family rented a car for 3 days. The family paid \$45.00 per day plus \$0.35 per km. The family travelled 327 km. How much did it cost to rent the car, before taxes?

6. The sum of a number and three more than the number is 23.
Let t represent the number.
Then, an equation is: $t + t + 3 = 23$
- Use algebra tiles to solve the equation. What is the number?
 - Verify your solution.



7. **Assessment Focus** Two times the edge length of a cube is 6 cm longer than the edge length.
Let l centimetres represent the edge length of the cube.
An equation for the edge length is: $2l = 6 + l$
- Use algebra tiles to solve the equation.
What is the edge length of the cube?
 - Verify the solution.
 - What are the surface area and the volume of the cube?

Take It Further

8. The sum of three consecutive numbers is 63.
- Write an equation you could use to solve this problem.
 - Solve the equation. What are the numbers?
 - Verify your solution.
9. Solve these equations. Verify your solutions.
- $7x + 4 = 3x - 8$
 - $3 - 2x = 13 + 3x$



Math Link

Science

Pressure is force per unit area.

Pressure is measured in pascals (Pa).

A formula for pressure is:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

When we know the pressure in pascals and the area in square metres, we can solve this formula to find the force in newtons (N).

Reflect

Explain how you can use algebra tiles to solve an equation with variables on both sides of the equal sign.
Include an example in your explanation.

Focus Solve a problem by solving a related equation.

Explore



Work with a partner. Solve this problem.

My mother's age is 4 more than 2 times my brother's age.

My mother is 46 years old.

How old is my brother?

Reflect & Share

Discuss the strategies you used for finding the brother's age with those of another pair of classmates.

Did you use an equation?

If not, how could you represent this problem with an equation?

Connect

In *Unit 1*, you learned how to solve equations algebraically.

All the equations in *Unit 1* had solutions that were whole numbers.

We use the same method to solve an equation where the solution is a fraction or a decimal.

Example 1

Three more than two times a number is 4. What is the number?

- Write an equation to represent this problem.
- Solve the equation.
- Verify the solution.

Solution

- Let the number be n .

Then, two times the number is: $2n$

And, three more than two times the number is: $3 + 2n$

The equation is $3 + 2n = 4$

- $$3 + 2n = 4$$

$$3 + 2n - 3 = 4 - 3$$

$$2n = 1$$

$$\frac{2n}{2} = \frac{1}{2}$$

$$n = \frac{1}{2}$$

To isolate $2n$, subtract 3 from each side.

Divide each side by 2.

Using the inverse operation here is the same as using zero pairs.

c) To verify the solution, substitute $n = \frac{1}{2}$ into $3 + 2n = 4$.

$$\begin{array}{l} \text{Left side} = 3 + 2n \\ \quad = 3 + 2\left(\frac{1}{2}\right) \\ \quad = 3 + 1 \\ \quad = 4 \end{array} \qquad \text{Right side} = 4$$

Since the left side equals the right side, $n = \frac{1}{2}$ is correct.
The number is $\frac{1}{2}$.

In *Example 1*, we could write the solution $n = \frac{1}{2}$ as a decimal,
 $n = 0.5$.

However, some fractions, such as $\frac{1}{3}$, are repeating decimals.
Do not convert a fraction of this type to a decimal.

We can use an equation to solve problems related to number
patterns.

When we know the n th term and the term value,
we can solve an equation to find the term number.

Example 2

The n th term of a number pattern is $5n - 2$.

What is the term number when the term value is 348?

Solution

The n th term is $5n - 2$.

The term value of an unknown term number is 348.

Write the equation: $5n - 2 = 348$

Solve this equation for n .

$$5n - 2 = 348$$

$$5n - 2 + 2 = 348 + 2$$

To isolate $5n$, add 2 to each side.

$$5n = 350$$

$$\frac{5n}{5} = \frac{350}{5}$$

Divide each side by 5.

$$n = 70$$

The 70th term has value 348.

In *Example 2*, the equation could have been solved by inspection:

$$5n - 2 = 348$$

Think: what do you subtract 2 from to get 348?

Answer: you subtract 2 from 350.

Think: what do you multiply 5 by to get 350?

Answer: you multiply 5 by 70.

So, $n = 70$

The equation could also have been solved by systematic trial:

$$5n - 2 = 348$$

Use a calculator to substitute different numbers for n until the left side of the equation equals 348.

In *Example 2*, there is only one value of n that makes the equation true. If $n = 69$, or if $n = 71$, or if n equals any number other than 70, the equation is not true.

Practice

Use algebra, systematic trial, or inspection to solve an equation.

- Solve each equation.
 - $2x = 3$
 - $3x = 2$
 - $4x = 6$
 - $5x = 12$
- Solve each equation. Verify the solution.
 - $2x - 1 = 5$
 - $7 = 1 + 3n$
 - $10 = 4a - 1$
 - $5 + 2m = 6$
- Write, then solve, an equation to answer each question. Verify the solution.
 - Ten more than three times a number is 25.
What is the number?
 - Ten less than three times a number is 25.
What is the number?
 - Twenty-five subtracted from one-half a number is 10.
What is the number?
 - One-half of a number is subtracted from 25.
The answer is 10.
What is the number?
- Navid has \$72 in her savings account.
Each week she saves \$24.
When will Navid have a total savings of \$288?
 - Write an equation you can use to solve the problem.
 - Solve the equation.
When will Navid have \$288 in her savings account?
 - How can you check the answer?



Calculator Skills

The area of a square is 225 m^2 .

Calculate the length of a diagonal of the square to the nearest centimetre.

Take It Further

Reflect

- 5. Assessment Focus** The Grade 8 students had an end-of-the-year dance. The disc jockey they hired charged a flat rate of \$85, plus \$2 for each student who attended the dance. The disc jockey was paid \$197. How many students attended the dance?
- Write an equation you can use to solve the problem.
 - Solve your equation. Verify the solution.
- 6.** The n th term of a number pattern is $4n - 3$.
- What is the term value for each term?
 - the 10th term
 - the 20th term
 - What is the term number for each term value?
 - 53
 - 97
- 7.** The n th term of a number pattern is $9n + 1$. What is the term number for each term value?
- 154
 - 118
 - 244
- 8.** Use this information:
Water flows into a bathtub at a rate of 15 L/min.
- Write a problem that can be solved using an equation.
 - Write, then solve, the equation.
- 9.** Use this information:
Boat rental: \$300 Fishing rod rental: \$20
- Write a problem that can be solved using an equation.
 - Write the equation, then solve the problem.
 - How could you have solved the problem without writing an equation? Explain.
- 10.** Two more than the square of a number is 123. What is the number?
- Write an equation you could use to find the number.
 - Solve the equation. What is the number?
 - Verify the solution.

Choose one of the word problems in this section. Explain the steps you used to write the equation, then to solve the equation.

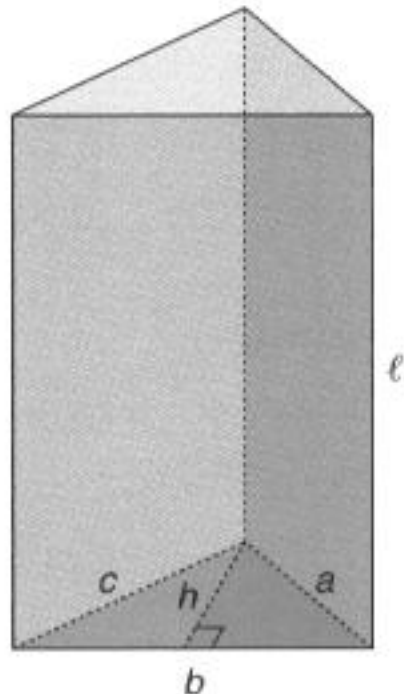
Writing a Journal

A journal is a place to record ideas, observations, illustrations, and responses.

The responses to *Reflect* in each lesson are often recorded in a journal.

Here are some ideas for other items to include.

- Comment on thoughts and feelings, successes and challenges:
 - I worked well in the group today because...
 - I could improve my skills with integer operations by ...
- Explain key math ideas, formulas, and words:
 - Write the word followed by a definition, picture, and example.Here is an example for *Unit 3*.

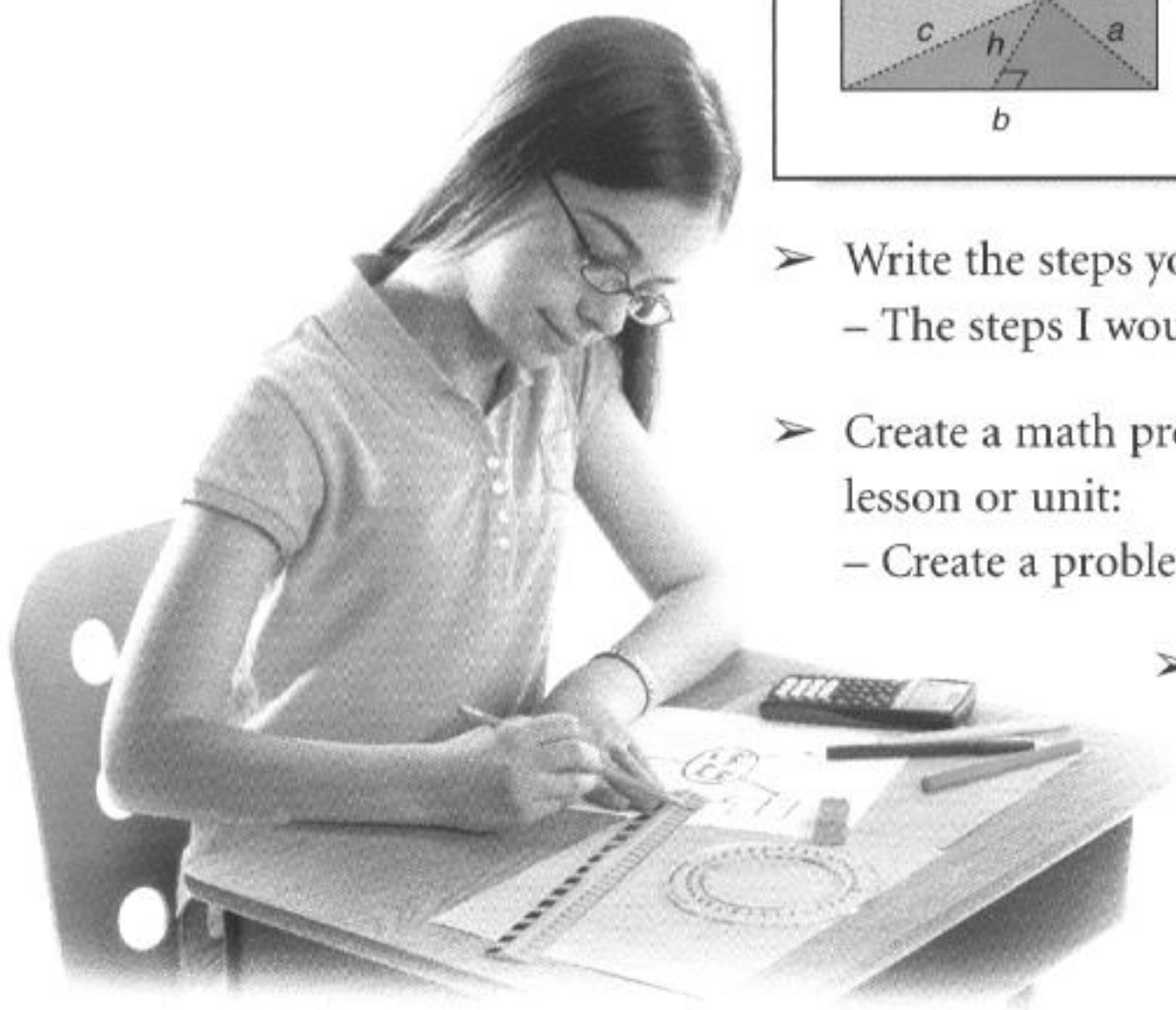


Triangular Prism

A triangular prism is a polyhedron with two congruent triangular bases, and its other faces are rectangles.

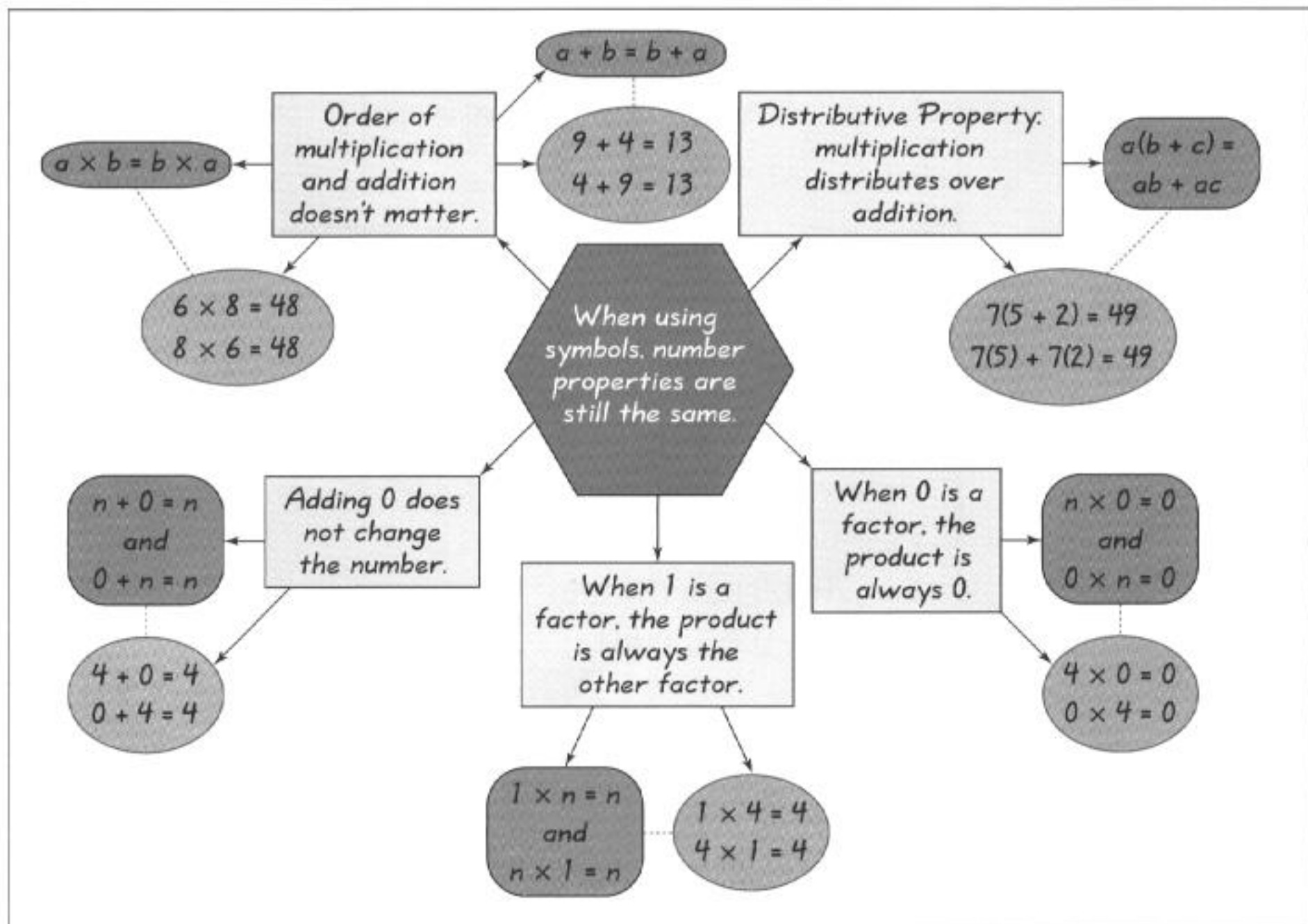
- The volume is:
 $V = \text{base area} \times \text{height}$
 $V = \frac{1}{2}bh\ell$
- The surface area is:
 $SA = \text{sum of the areas of the faces}$
 $SA = al + bl + cl + bh$

- Write the steps you would use to do a math task:
 - The steps I would follow to draw a circle graph...
- Create a math problem that uses the ideas from the lesson or unit:
 - Create a problem you could use an equation to solve.
- Make a list of examples of a math topic. Use headings to organize the list:
 - List the different types of problems that involve percents.
 - Draw different kinds of polygons with the same attributes.



- Explain or justify a solution, pattern, or choice of strategy:
 - This solution makes sense because...
 - I chose to make a model because ...
- Explain how you could apply the math:
 - Who would need to calculate the area of a circle? Why?
 - How does the media use charts and graphs when they want to persuade?
 - Math was in the news today...
- Summarize what you learned:
 - The main ideas I learned today (this week) are...
 - Draw a concept map to show the key ideas today (this week).

Here is a concept map for *Unit 10, Lesson 10.1*.





Health Care Professionals

In the 16th century, a person who needed medical attention often went to the local hair-cutting shop for treatment. The patient would be seen by a *barber-surgeon*, someone who was not only skilled at cutting hair, but also trained to cut into the human body. In fact, a surgeon was often nicknamed “sawbones.” The tool most often used by a barber-surgeon was a leech!

Today, hospitals and emergency rooms are staffed with medical experts. From the paramedic who may treat the patient on the way to the hospital, to the nurse in the recovery ward, everyone has extensive training. Mathematics is an important part of this training.

Suppose a doctor prescribes a patient 30 mg of a certain drug. The medicine is in a bottle, with 150 mg of the drug diluted in 20 mL of liquid. How many millilitres of the medicine must the nurse give the patient? The nurse must be precise because too much or too little of the drug could harm or even kill the patient. The nurse uses this equation to determine the dosage, in millilitres:

$$\begin{aligned}\text{Dosage} &= \frac{\text{amount of drug needed}}{\text{amount of drug diluted in bottle}} \times \text{the amount of liquid in bottle} \\ &= \frac{30}{150} \times 20 \\ &= \frac{1}{5} \times 20 \\ &= 4\end{aligned}$$

The correct dose is 4 mL.

Calculating dosages for children is often based on their body mass, and the dosage will be a fraction of a typical adult dose.

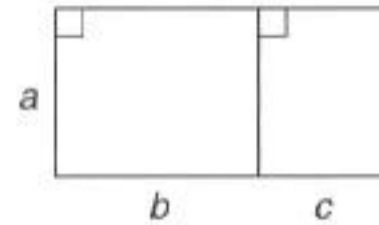


What Do I Need to Know?

Distributive Property

The product of a number and the sum of two numbers can be written as a sum of two products:

$$a(b + c) = ab + ac$$



The n th Term of a Number Pattern

- The n th term can be used to find the term value of any term in a pattern.

For example, for the pattern with n th term $3n + 2$, the 9th term is: $3(9) + 2 = 29$

- The n th term can also be used to find the term number when the term value is known.

For example, for the pattern with n th term $3n + 2$, to find the term number with the term value 23, solve the equation $3n + 2 = 23$, to get $n = 7$.

The 7th term has value 23.

What Should I Be Able to Do?

For extra practice, go to page 497.

LESSON

10.1 1. Expand.

- $6(x + 9)$
- $3(11 + 4x)$
- $5(7x + 6y + 5)$
- $4(3a + 5b + 7c)$

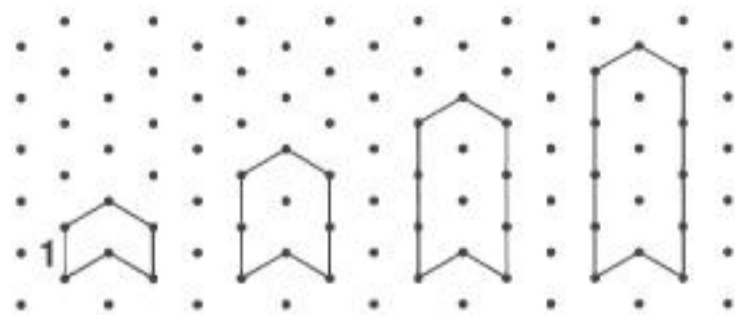
10.2 2. For each algebraic expression, substitute $n = 1, 2, 3, 4,$ and 5 to generate a number pattern. Describe each pattern, then write a pattern rule.

- $3n + 5$
- $5n + 15$

3. For each number pattern below:

- Write a pattern rule. Justify your rule.
- Graph the pattern. Use the graph to find the 7th term.
- Write an expression for the n th term.
- Find the 70th term.
 - $8, 12, 16, 20, 24, \dots$
 - $5, 7, 9, 11, 13, \dots$

- 10.3 4. Here is a pattern drawn on isometric dot paper.



Frame 1 Frame 2 Frame 3 Frame 4

The distance between two adjacent dots is 1 unit.

The pattern continues.

- Find the perimeter of each frame. What pattern do you see in the perimeters?
 - Use a pattern to find the perimeter of Frame 9.
 - Write an expression for the perimeter of the n th frame.
 - Find the perimeter of Frame 50.
- 10.4 5. Interpret each equation in words. Then use algebra tiles to solve the equation. Verify each solution.
- $12 - x = 3x$
 - $4x - 7 = 2x + 3$
 - $3x - 8 = x$
 - $3 - 7x = 7 - 9x$
6. Five less than two times a number is equal to one less than the number. Let n represent the number. Then, an equation is:
 $2n - 5 = n - 1$

- Use algebra tiles to solve the equation.
What is the number?
- Verify your solution.

- 10.5 7. Solve each equation. Verify the solution.
- $3x + 2 = 4$
 - $4x = 10$
 - $11 = 3x + 1$
 - $4x - 7 = x + 1$
8. The school's sports teams hold a banquet. The teams are charged \$125 for the rental of the hall, plus \$12 for each meal served. The total bill was \$545. How many people attended the banquet?
- Write an equation you could use to solve the problem.
 - Solve your equation.
 - Verify the solution.
9. The n th term of a number pattern is $4n - 1$.
- Write the first 5 terms of the pattern.
 - Which term number has each term value?
 i) 79 ii) 139 iii) 395
10. a) Write an expression for the n th term of this number pattern:
 7, 13, 19, 25, ...
- Use the expression in part a. Which term number has each term value?
 i) 151 ii) 307 iii) 433

Practice Test

1. Interpret each equation in words.
Solve the equation.
Verify the solution.
 - a) $x + 5 = 3x - 9$
 - b) $2x - 5 = 10$
2. Whoopi saves pennies. She has 10¢ in her jar at the start. Whoopi starts on January 1st. She saves 3¢ every day.
 - a) How many pennies does Whoopi have in the jar on each of January 1st, 2nd, 3rd, 4th, 5th, and 6th?
Record the results in a table.
What pattern do you see? Write a pattern rule.
 - b) Write an expression for the n th term.
 - c) Use the expression to find the 25th term.
 - d) How could you find how much money Whoopi saved in January?
3. Anoki is holding a skating party.
The rental of the ice is \$75, plus \$3 per skater.
 - a) Write an expression for the cost in dollars for n skaters.
 - b) Use the expression in part a to find the total cost for 25 skaters.
 - c) What if Anoki has a budget of \$204. Write an equation you can solve to find how many people can skate.
Solve the equation.
4. Two number patterns have these n th terms.
Pattern A: $6n + 4$
Pattern B: $5n - 3$
 - a) Find the 48th term of Pattern A.
 - b) Use the term value from part a.
Which term number in Pattern B has this term value?
How do you know?

Suppose your older sister has bought a cell phone. She asks for your help to find the best cell phone plan.

Part 1

- Here are three cell phone plans.
Each plan includes 200 free minutes.

CanTalk: \$30.00 per month, plus \$0.30 per additional minute

Connected: \$35.00 per month, plus \$0.25 per additional minute

In-Touch: \$40.00 per month, plus \$0.20 per additional minute

Copy and complete this table.

Number of Additional Minutes	40	80	120	160	200
CanTalk					
Connected					
In-Touch					

- Which plan would you choose if your sister uses 40 additional minutes per month? 120 additional minutes? 200 additional minutes? Explain.
- Graph the data in the table.
Use a different colour for each plan.
Join each set of points with a broken line.
Label each line with the name of the plan.
What patterns do you see?
What happens to the lines when the number of additional minutes is 100?
What does this represent?
Which plan would you choose if your sister uses 100 additional minutes per month? Explain.



Convert each money amount to cents before you write the equations.

Check List

Your work should show:

- ✓ all tables and graphs, clearly labelled
- ✓ the expressions and equations you wrote, and how you used them to solve the problems
- ✓ detailed, accurate calculations
- ✓ clear explanations of your solutions and the patterns you observed

Part 2

For each plan, write an expression for the monthly cost of n additional minutes.

Use each expression to find the total monthly cost for 85 additional minutes for each plan.

Suppose your sister can spend \$80 a month on her cell phone.

Write an equation you can solve to find how many additional minutes she can afford with each plan.

Solve each equation. Explain what each solution means.

Part 3

Write a paragraph to explain what decisions you have made about choosing the best cell phone plan.



Reflect on the Unit

Explain how patterns, expressions, and equations are used to solve problems. Include an example in each case.